On the Use of Regulator Theory in Neuroscience with Implications for Robotics

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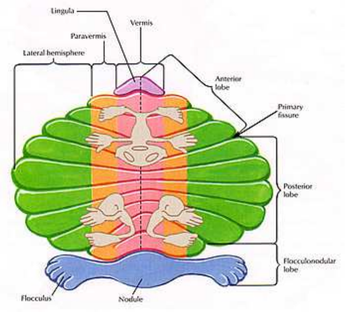
July 8, 2021

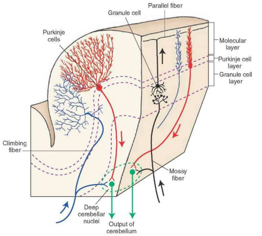
Neuroscience

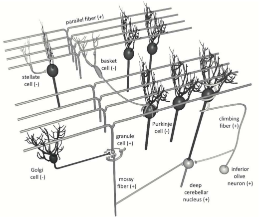
The Cerebellum



- The **cerebellum** is involved in regulation of eye movement, gait, posture, fine movements, emotion regulation, language, attention, etc. Namely, all behaviors requiring precision control.
- It consists of three major subdivisions: **vestibulocerebellum**, **spinocerebellum**, and **cerebrocerebellum**.
- It can be further subdivided into independent **functional modules**.
- The **cerebellar microcircuit** is identical in each functional module, leading to the belief that the cerebellum performs a **universal computation**.

Cerebellar Microcircuit





Diedrichsen and Bastian. MIT Press 2014.

- **J. Eccles, M. Ito, and J. Szentagothai**. *The cerebellum as a neuronal machine.* 1967.
- Information flows from **mossy fiber** inputs (MFs) to **granule cells**, which act as filters, then via **parallel fibers** (PFs) to **Purkinje cells** (PCs), whose axons form the only output pathway to one of the **deep cerebellar nuclei**.
- Each functional module processes its own **sensory error signal** received via **climbing fiber** (CF) inputs.
- Adaptive capability is provided by CFs which change the synapse strength between PFs and PCs.

(v) Each deep cerebellar nucleus has a projection to the MF inputs, called the **nucleo-cortical pathway**.

Structural Model

At a systems or structural level,

$$
\begin{aligned}
\dot{x} &= Ax + Bu + Ed_1 && \text{(1a)}\\
e &= Cx + Dd_2 && \text{(1b)}\\
\dot{w}_1 &= F_1 w_1 + G_1 u_{mf,1} && \text{(1c)}\\
&\vdots \\
\dot{w}_k &= F_k w_k + G_k u_{mf,k} && \text{(1d)}\\
\dot{w}_{k+1} &= F_{k+1} w_{k+1} + G_{k+1} u_{im} && \text{(1e)}\\
\hat{w} &= (w_1, \dots, w_{k+1}) && \text{(1f)}\\
\dot{\hat{\Psi}} &= \gamma e \hat{w}^{\mathrm{T}} && \text{(1g)}\\
u_{im} &= \hat{\Psi} \hat{w} && \text{(1h)}\\
u &= u_s + u_{im}\,. && \text{(1i)}
\end{aligned}
$$

- $e \in \mathbb{R}$ is the **sensory error**, $d_1, d_2 \in \mathbb{R}$ are **disturbance signals**.
- Filter (1e) models the **nucleo-cortical pathway**.
- (1g) models the modifiable synapses between PFs and PCs.
- This model resembles an **adaptive filter**. Neuroscientists posit it models the plant being regulated (e.g. eye, arm, leg, etc).

Cerebellar Function

- Behaviors such as: *visuomotor adaptation; arm reach in a force field; walking in snow; standing on a lurching subway train; lifting a heavy object; reaching the arm in water with the vision distorted; making saccades to a moving target; smoothly tracking a moving target with the eyes; holding the eyes at an eccentric position; interaction forces between limbs*; etc, are all manifestations of **disturbance rejection**.

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- We **hypothesize** that the cerebellum is involved in all behaviors of precision control because it is the *unique, centralized, and universal disturbance rejection unit* of the brain at the behavioral level.

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- Further, the cerebellum performs disturbance rejection with insufficient sensory measurements. Despite a paucity of incoming information, reference and disturbance signals must be rejected for smooth motor and cognitive function.

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- We **hypothesize** that the cerebellum is involved in all behaviors of precision control because it is the *unique, centralized, and universal disturbance rejection unit* of the brain at the behavioral level.
- Further, the cerebellum performs disturbance rejection with insufficient sensory measurements. Despite a paucity of incoming information, reference and disturbance signals must be rejected for smooth motor and cognitive function.
- The branch of control theory that deals with rejecting unmeasurable, exogenous reference and disturbance signals is **regulator theory**. It's main tenet is the **internal model principle of control theory**.
- The principle states that *any good controller must contain a model of all persistent, exogenous reference and disturbance signals entering into a control loop*.

Internal Model Principle in Neuroscience

- By the 1960's it was known the human eye could track predictable moving targets with near zero steady-state error despite > 100ms delay of the retinal error signal.
- **Humans use internal models to estimate gravity and linear acceleration.** Merfeld et. al. *Nature*, 1999.
- **Does the brain model Newton's laws?** McIntyre et. al. *Nature*, 2001.
- **Neurons compute internal models of the physical laws of motion.** Angelaki, et. al. *Nature*, 2004.
- **An internal model of a moving visual target in the lateral cerebellum.** N. Cerminara, R. Apps, and D. Marple-Horvat, 2009.
- **Internal models of eye movement in the floccular complex of the monkey cerebellum**. S. Lisberger. *Neuroscience*, 2009.
- **Consensus paper: the role of the cerebellum in perceptual processes.** K. Cullen. *Cerebellum*, 2015.
- **A unified internal model theory to resolve the paradox of active versus passive self-motion sensation**. J. Laurens and D. Angelaki. *eLife*, 2017.

Control Theory

Adaptive Internal Models

- Consider an open-loop system

  $$\begin{aligned} \dot{x} &= Ax + Bu + Bd \\ e &= Cx\,, \end{aligned}$$

  where $x \in \mathbb{R}^n$, $e, u, d \in \mathbb{R}$.
- We assume disturbances d are modeled by a linear **exosystem**

  $$\dot{w} = Fw + Gd\,, \quad d = \Psi w$$

  where $w \in \mathbb{R}^q$, and exosystem parameters $\Psi \in \mathbb{R}^{1\times q}$ are unknown.
- Suppose x is measurable and (A, B, C) are known. An **adaptive internal model** is

  $$\begin{aligned} \dot{w}_0 &= Fw_0 + FNx && \text{(2a)} \\ \dot{w}_1 &= Fw_1 - NAx && \text{(2b)} \\ \dot{w}_2 &= Fw_2 - NBu && \text{(2c)} \\ \hat{w} &= w_0 + Nx + w_1 + w_2 && \text{(2d)} \\ \dot{\hat{\Psi}} &= \gamma(B^{\mathrm{T}}Px)\hat{w}^{\mathrm{T}} && \text{(2e)} \\ u_{im} &= -\hat{\Psi}\hat{w} && \text{(2f)} \\ u &= u_s + u_{im}\,, && \text{(2g)} \end{aligned}$$

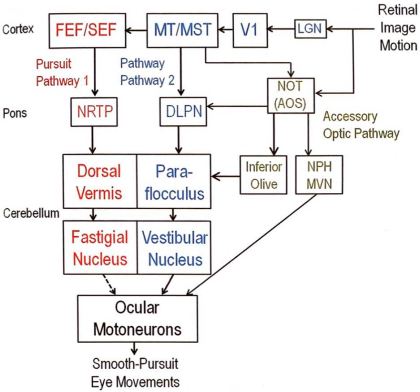
  where $NB = G$, $\hat{\Psi}$ is an estimate of Ψ, and $\gamma > 0$ is the adaptation rate.
- Filter (2c) ensures the internal model principle will be satisfied.

Slow Eye Movement Systems

Oculomotor System

Premise: *For clear vision, objects must be positioned stably on the fovea.*

- **Vestibulo-ocular reflex**
- **Gaze Holding**
- **Smooth Pursuit**
- **Optokinetic system**
- **Saccadic system**
- **Vergence system**



Leigh and Zee. Oxford Univ. Press 2015.

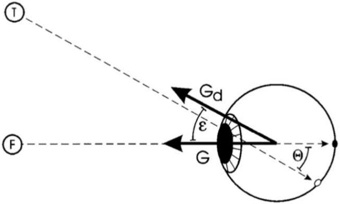
Open-loop Model

- We consider only horizontal motion of one eye. The **eye position** x is relative to the head. The **gaze** is $x + x_h$.
- The first-order model of the **oculomotor plant** is

$$\dot{x} = -K_x x + u\,,$$

  with time constant $1/K_x \simeq 200$ ms.
- The **retinal error** is

$$e := \alpha_e(r - x_h - x)\,.$$



Crawford and Guitton. J. Neurophys. 1997.

Assumptions.

(i) $\alpha_e = 1$.

(ii) *There is no proprioception of eye position in the brain.*

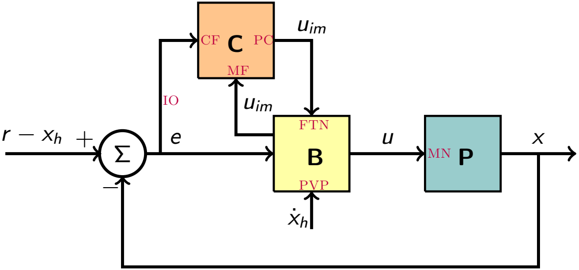
Slow Eye Movement Model

The overall model of the oculomotor system is:

$$\begin{aligned}
\dot{x} &= -K_x x + u \\
\dot{\hat{x}} &= -K_x \hat{x} + u \\
\dot{\hat{w}} &= F\hat{w} + G(u_s + u_{im}) \\
\dot{\hat{\Psi}} &= e\hat{w}^{\mathrm{T}} \\
u_b &= \alpha_x \hat{x} - \alpha_h \dot{x}_h \\
u_s &= K_e e \\
u_{im} &= \hat{\Psi}\hat{w} \\
u &= u_b + u_s + u_{im}\,.
\end{aligned}$$

[B20A] M. Broucke. Model of the oculomotor system based on adaptive internal models. IFAC World Congress, July 2020.

Control Architecture



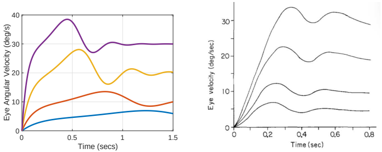
[B21] M. Broucke. Adaptive internal model theory of the oculomotor system and the cerebellum, IEEE TAC, 2021.

Summary of Simulations

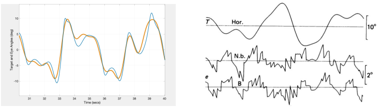
- We recover **behavioral**, **neurological**, and **lesion** experiments, both steady-state and transient response.

VOR	**Smooth Pursuit**
VOR with sinusoidal head rotation	*One sinusoid, Lisberger, 2009*
Short-term adaptation of VOR	*Ramp tracking, Robinson, 1986*
Ramp in head position, Lisberger et.al. 1986	*Error clamp, Stone & Lisberger, 1990*
PC firing, Lisberger & Fuchs, 1978	*Target stopping, Krauzlis & Miles, 2006*
VOR gain in dark, Robinson, 1981	*Four sinusoids, Collewijn & Tamminga, 1984*
VOR gain in light, Miles & Eighmy, 1980	*Time delay*
VOR cancellation, Buettner & Buttner, 1979	*Perturbations, Lisberger, 2010*
Cerebellectomy, Zee, 1981	*SP with adapted VOR gain, Lisberger 1994*
VOR w/ NPH lesion, Kaneko, 1991	*SP w/ NPH lesion, Kaneko, 1999*
Visual-vestibular conflict, Waespe & Henn, 1978	
Gaze Holding	
PC output during gaze holding, Noda & Suzuki, 1979	
NPH lesion, Skavenksi & Robinson, 1973	

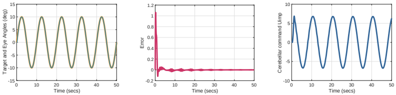
Smooth Pursuit



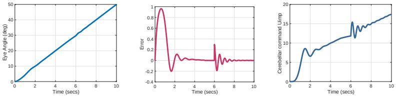
Robinson et.al. Biol. Cyb. 1986



Collewijn and Tamminga. J. Physiology 1984



Smooth pursuit with 107ms time delay



Cerminara et.al. J. Physiology 2009

Visuomotor Adaptation

Open-loop Model

- **Visuomotor adaptation** is a subconscious, "machine-like" brain process taking place over repetitive trials, elicited by a visual error closely following the execution of a movement, and intended to calibrate over a lifetime the mapping between what is seen and how to move.
- Consider a single degree of freedom. The scalar **open-loop model** is

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ e(k) &= r(k) - x(k) - d(k)\,, \end{aligned}$$

where k is the **trial number**, $x(k)$ is the **position** (or angle) of that degree of freedom, $d(k)$ is a visual **disturbance**, $r(k)$ is a desired **target position** (or angle), and $e(k)$ is the visual **error**, also the measurement.

Assumptions.

- *$A = 0$ (no proproception, i.e. no "self information").*
- *For the sake of simplicity, let $B = 1$.*
- *d is a predictable disturbance that can be modeled by a linear exosystem.*

Model of Visuomotor Adaptation

- The **open-loop model** (again) is

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ w(k+1) &= Fw(k) + Gd(k)\,, \qquad d(k) = \Psi w(k) \\ e(k) &= r(k) - x(k) - d(k)\,. \end{aligned}$$

- The **adaptive internal model** is

$$\begin{aligned} w_0(k+1) &= Fw_0(k) + FGe(k) \\ w_1(k+1) &= Fw_1(k) - Gu(k) \\ \hat{w}(k) &= w_0(k) + Ge(k) - w_1(k) \\ u(k) &= u_s(k) + u_{im}(k) = Ke(k) + \Psi \hat{w}(k)\,, \end{aligned}$$

where w_0, w_1 are filter states and $\hat{w}(k)$ is an estimate of $w(k)$.

[GB20] A.A. Gawad and M.E. Broucke.
Visuomotor adaptation is a disturbance rejection problem.
IEEE Conference on Decision and Control. 2020.

Simulations

- The model recovers the standard behaviors of visuomotor adaptation: **savings**, **reduced savings**, **anterograde interference**, **rapid unlearning**, **rapid downscaling**, and **spontaneous recovery** *(Smith et.al. PLoS Comp. Bio. 2006).*

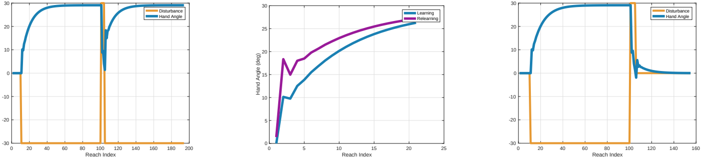


Figure 5 : Savings in the left two figures. Anterograde interference in the right figure.

- These behaviors are sometimes characterized as forms of *learning*, but our work suggests they may be interpreted as a more naive, low level process.

Implications for Robotics

Robot Tool Learning

- A robotic manipulator equipped with **foveated, movable cameras** is capable to perform rapid reaching movements to targets. It is tasked with using a new tool, inducing an **offset** d of its end effector.
- N **targets** are positioned in the robot's visual field. Let $r(i)$ denote the horizontal angular position of the ith target.
- Each target i has associated to it a **feedforward motor command** $u_{f,i}$, acquired through prior experience.
- The visual field is partitioned into **adaptation fields**. Each adaptation field has associated to it an adaptive internal model.
- The index of the target foveated at the end of the kth reach is $m(k) \in \{1, \ldots, N\}$. The robot only records error measurements for targets it has foveated on.
- The index of the target for the $(k+1)$th reach is $t(k) \in \{1, \ldots, N\}$.

Robot Tool Learning

- The open-loop model is:

$$\begin{aligned} x(k+1) &= u(k) \\ e(k) &= r(m(k)) - x(k) - d(k)\,. \end{aligned}$$

- For $i = m(k)$, the foveated target:

$$\begin{aligned} w_{0,m}(k+1) &= Fw_{0,m}(k) + FGe(k) \\ w_{1,m}(k+1) &= Fw_{1,m}(k) + G(u(k) - u_{f,m}) \\ \hat{w}_m(k) &= w_{0,m}(k) + Ge(k) - w_{1,m}(k) \\ u(k) &= u_{f,t} + \psi\hat{w}_t(k)\,, \end{aligned}$$

  where $m = m(k)$ and $t = t(k)$.

- For $i \neq m(k)$:

$$\begin{aligned} w_{0,i}(k+1) &= F_n w_{0,i}(k) \\ w_{1,i}(k+1) &= F_n w_{1,i}(k) \\ \hat{w}_i(k) &= w_{0,i}(k,i) - w_{1,i}(k) \end{aligned}$$

  $F_n = 0.999$, i.e. internal model i slowly dissipates its state.

Robot Tool Learning

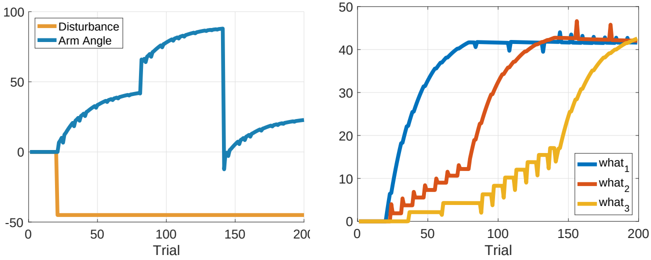


Figure 6 : A robot arm reaching for each of three targets, while occasionally glancing at the other two.

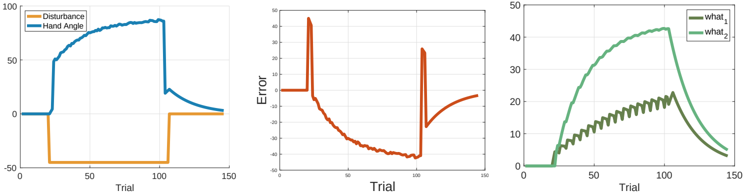


Figure 7 : Mazzoni and Krakauer's experiment.

Conclusions

- A mathematical model of the cerebellum would improve the understanding of brain diseases such as ataxia and Parkinson's. And it would reduce the reliance on open-brain monkey experiments.
- We have investigated the slow eye movement systems, the optokinetic system, and visuomotor adaptation. Other motor systems such as balance and gait must be explored.
- Further developments in control theory on adaptive internal models are needed to address the requirements of biological systems.
- We expect future humanoid robots will be equipped with **cerebellar intelligences** such as visuomotor adaptation, reflex adaptation, and other behaviors still to be identified.

Thank You!